Frequency-Independent Scattering Solutions for Simple Geometries

Clayton Davis and Weng Cho Chew

1. Introduction

A natural approach for solving large, high-frequency scattering problems is to augment a numerical (simulated) solution with the notion that light travels as rays. This combines the robustness of numerical methods, where every detail of the problem is modeled in the computer, with the quick time-to-solution characteristic of ray models. The resulting algorithms have potential to reduce overall computational cost, and in some cases, give costs that are frequency independent. Mathematically, one says that the current solution decomposes into the sum of slowly varying amplitude functions modulating rapidly oscillating phase factors. Since the amplitude functions vary slowly, they are more amenable to discretization than the total current.

Decomposing the total current in this way introduces a host of technical difficulties. Solving the general problem would require a gargantuan effort; hence, all research to date solves specific, simple problems for which the analysis is more tractable. Here, we focus on TEz scattering from perfectly conducting (PEC) scatterers composed of flat facets.

2. Frequency-Independent Formulation

In general, the current on an arbitrary scatterer may be written as a sum of amplitude functions $J_p$ with associated wavevectors $k_p$:

$$J(r) = \sum p J_p(r) e^{ik_p \cdot r}$$

*Equation 1*

The formulation (1) shifts the computation burden from the number of nodes required to resolve the current (since the amplitude functions are slowly varying) to the number of integration points required to fill the moment matrix. Hence, to solve the ansatz (1) using the method of moments, a frequency-independent rule is required to compute the associated moment matrix interactions, which oscillate rapidly due to the phase term and the natural, rapid oscillation of the Green’s function for large $k$.

The numerical method of steepest descent (NSD) has been proposed recently by [1] and [2]. We advocate the method of steepest descent in view of its accuracy and efficiency when compared to alternative approaches. For instance, although the method of stationary phase has been successfully employed [3], it requires the user to specify both the number of quadrature points and the width of a truncating window about the stationary phase point [4]. It is more straightforward to truncate the infinite integrals that arise in the method of steepest descent. Furthermore, it is observed that a very wide window is required, so that the benefits of the stationary phase method are not realized until extremely high frequencies. We also note that a logarithmic-cost algorithm is given by [5] for wave boundary elements and by [6] for physical optics integrals.

From a practical point of view, two issues still remain from previous work: First, the complexifying of the field and/or source points breaks down in many situations. For instance, the Jacobian of the
transformation can become peaked, even infinite. Though these issues can be avoided by restoring the complex-valued nodes to the real line, doing so requires the coding of a very large number of "cases," and generating a robust algorithm is difficult. It is apparently for this reason that only specific examples are examined in [1], where the NSD can be made to work. This also seems to be a motivating factor behind [7], where a coordinate change is made so that the NSD does not break down, but the method fails to be frequency independent because the transformation cannot be applied throughout the entire region of integration.

In short, an integration routine with cost independent of frequency may be obtained by making integration variables complex-valued, but no general, robust algorithm has been demonstrated to date. The singularity of the Green's function is a second source of frequency-dependent cost. A special change of variables not discussed here is required to perform singular integrations.

It is not possible to provide all the mathematical details of NSD here, and the reader is referred to [8] and [2] for the full problem solution. Here, we only demonstrate the utility and robustness of this routine. The results are based on the geometry of Figure 1, where the field facet (length \( l \)) remains fixed. The angles \( \psi, \phi_p, \phi_q \), and the source length \( l' \) are varied to probe the quadrature rule for robustness with respect to changing geometric parameters. The field to source separation is \( 10l \). All errors shown are for the evaluation of

\[
\int dr \int dr' \frac{e^{ikR-it\tau} - e^{-ikR-it\tau}}{R} \]

Equation 2

The factor of \( 1/R \) is for convenience in generating the reference solution.

To generate this result, Equation (2) is evaluated (at each frequency) for \( 10^4 \) combinations of different \( \psi, \phi_p, \phi_q \), and \( l' \), where each parameter is varied over the ranges \( \pi/10 < \psi < 9\pi/10 \), \( 0 < \phi_{p,q} < 2\pi \), and \( l/10 < l' < 10l \). The maximum error in each set of \( 10^4 \) data points is shown, and it is clearly bounded, at least for the data ranges shown. This is achieved with a bounded number of quadrature points (10 and 20 points per complex-valued path) for all frequencies.

![Figure 1. Geometric parameters used to test the robustness and frequency independence of the NSD formulation presented here.](image)
3. High-Frequency Scattering Example

In this section, we consider a single scattering problem, with more examples given in [8]. The test solution is generated using the method of moments and a three-term ansatz (1), with one amplitude function corresponding to physical optics, and two amplitude functions for diffraction. The quadrature rule uses 20 points per path, and was previously demonstrated to be robust and accurate even at very high frequencies. The discretization uses 28 nodes per amplitude function per scatterer surface, which amounts to 57 total unknowns per amplitude function. The scatterer consists of two flat faces that meet at an interior angle of $157.5\degree$. The geometry and incident wave vector are illustrated in Figure 3, where it is seen that the total length of the faces is $120\lambda$. Continuity of $\mathbf{J}$ at the corner is enforced by the piecewise linear basis functions.

Figure 4 shows good agreement between the test (ansatz-based) current solution and an independent reference. Because the oscillation is very rapid, Figure 4 also shows windowed views of the current in the lower-left edge of the figure, which corresponds to the left edge of the horizontal scatterer face.
Similarly, the right side of Figure 4 corresponds to the upper-right edge of the slanted face in Figure 3.

![Figure 4](image-url)

**Figure 4:** Total current using the method of moments with frequency independent cost (red) and an independently-generated reference (blue). Agreement is good everywhere.

The far-field pattern is shown in Figure 5. Because very rapid variations in the far-field pattern are not always meaningful, we average the far-field pattern over $5^\circ$ increments. This has the physical interpretation of time-average power radiated through each angular region, far from the scatterer. Averaging for the test solution uses 100-point Monte Carlo integration, and the reference pattern is averaged brute-force. Test and reference far-field patterns are visually indistinguishable, similar to the accuracy obtained for the current solution. The averaged physical optics (PO) solution is also shown. The PO solution agrees well with the numerical solutions in the forward scattering direction ($60^\circ$) and in the two specular directions ($300^\circ$, $345^\circ$).

![Figure 5](image-url)

**Figure 5:** Far-field pattern corresponding to the current in Figure 4, where the pattern has been averaged in $5^\circ$ increments. Test and reference solutions are visually indistinguishable. The physical optics solution is also presented, and agrees with the numerical solution in the forward and specular directions.
REFERENCES


